# Infrared Hierarchy, Thermal Brane Inflation and Superstrings as Superheavy Dark Matter

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#### Abstract

In theories with TeV scale quantum gravity the standard model particles live on a brane propagating in large extra dimensions. Branes may be stabilized at large (sub-millimeter) distances from each other, either due to weak Van der Waals type interactions, or due to an infrared analog of Witten's inverse hierarchy scenario. In particular, this infrared stabilization may be responsible for a large size of extra dimensions. In either case, thermal effects can drive a brief period of the late inflation necessary to avoid the problems with high reheating temperature and the stable unwanted relics. The main reason is that the branes which repel each other at zero temperature can be temporarily glued together by thermal effects. It is crucial that the temperature needed to stabilize branes on top of each other can be much smaller than the potential energy of the bound-state, which drives inflation. After 10-15 e-foldings bound-states cool below the critical temperature and decay ending inflation. The parallel brane worlds get separated at this stage and superstrings (of a sub-millimeter size) get stretched between them. These strings can have the right density in order to serve as a superheavy dark matter.

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#### 1 Introduction.

It was suggested recently, that the fundamental scale of quantum gravity may be as low as TeV, provided there are N large new dimensions to which gravity can propagate[1]. The relation between the observed Planck scale  $M_P$  and the fundamental one M is then given by

$$M_P^2 = M^{N+2} V_N, \tag{1}$$

where  $V_N \sim R^N$  is the transverse volume of extra space. In this picture, all the standard model particles must live in a brane (or a set of branes) with 3 extended space dimensions. <sup>2</sup>. The supersymmetry breaking in the observable world may then result from a non-BPS nature of our brane Universe[7].

Perhaps the most natural realization of this picture is via the D-brane constructions (see[8] for an introduction). The standard model fields can be identified with the open string modes stuck on a D-brane, whereas gravity comes from the closed string sector propagating in the bulk [2, 9, 10].

Cosmological constraints discussed in[11], suggest that in such a scenario there is an absolute bound on the reheating temperature of the Universe due to the over-production of the bulk Kaluza-Klein gravitons. In particular, for N=2 this bound gives  $T_R \leq T_* \sim \text{MeV}$ , and even for larger N,  $T_*$  stays well below TeV. Obviously, with such a low reheating temperature it is hard to accommodate conventional four-dimensional scenarios for inflation or baryogenesis[12].

However, brane picture opens up the new, intrinsically high-dimensional mechanisms, both for inflation[13] and for baryogenesis[14]. Baryon asymmetry for instance, may result because of the inflow of the baryonic charge to our brane world, due to creation of the baby branes, or due to baryon number transport in the brane-brane collision[14]. Such a scenario of the baryogenesis does not necessarily require high reheating temperature. In fact, the temperature of the inflaton decay products, after they thermalize, can be lower that the typical baryon mass.

Recently a new inflationary mechanism, brane inflation, has been proposed [13]. Inflation is driven by the displaced branes that slowly fall on top of each other. This slow-fall in due to the weak inter-brane attraction and translates as slow-roll of the inflaton in an effective four-dimensional field theory language. Inflation ends by the brane "collision" which reheats the Universe. In its most straightforward version this scenario may suffer from a high reheating temperature problem  $(T_R \sim M)$ , as in particular was pointed out by Banks, Dine and Nelson[15].

In the present paper we will argue that branes may provide a built-in mechanism for avoiding this problem. In fact, the same high reheat temperature can trigger a secondary stage of the brief brane inflation with a very low reheating temperature.

<sup>&</sup>lt;sup>2</sup>The attempt of lowering the Planck scale to  $M_{GUT} \sim 10^{16} \text{GeV}$  goes back to[3]. In a different context lowering the string scale to TeV, without lowering the fundamental Planck scale was suggested in [4]. Dynamical localization of the fields on a (solitonic) brane embedded in a higher dimensional universe has been studied earlier in the field theoretic context[5], [6]

We show that this scenario can have an interesting byproduct, producing superstrings (of sub-millimeter length), which can serve as a superheavy dark matter!

Our scenario can be summarized as follows. In the absence of supersymmetry, branes are not BPS states and experience Van der Waals type interactions due to exchange of the light bulk modes (e.g. graviton, dilaton and Ramond-Ramond (RR) fields). Depending on the balance between the repulsive and attractive messenger forces, the branes can repel each other at the short distances and get stabilized at some large distance from one another. Alternatively, a large distance stabilization can be achieved via the analog of Witten's inverted hierarchy scenario[16]. This way of generating the large inter-brane separation can provide an explanation for the large size of extra dimensions, without any reference to big input quantities! (The alternative way would be to postulate some big conserved numbers, e.g. such as the number of branes, or a topological charge of the Universe [18, 19]).

However, whatever source stabilizes branes at zero temperature, the thermal effects change the picture. At high temperature, some branes that would normally repel each other at zero temperature, get stabilized on top of each other. In the other words at high T coincident branes correspond to a meta-stable minimum of the free energy.

The crucial point is that the temperature required for their stabilization can be much smaller than the potential energy of the boundstate. This energy drives a brief period of inflation, with a very low reheating temperature, just enough to get rid of unwanted relics. When the temperature drops to a certain critical value  $T_c$ , the boundstate becomes unstable and branes roll away marking the end of inflation. During this process strings get stretched between branes, which can serve as a new source of a superheavy dark matter. After rolling away brane-brane bound system oscillates around the equilibrium point and reheats the Universe to an acceptably low temperature.

Before proceeding, we want to note that density fluctuations will not be discussed in the present paper. We'll assume that these fluctuations are created at the earlier stage (e.g. by an earlier radius inflation [20, 21]).

#### 2 Van der Waals Forces Between the Branes

Let us consider the two parallel branes in a space of N > 1 transverse dimensions. Assume that the distance between the branes (r) is much larger than their "thickness", typically given by an inverse scale of tension  $(\sigma)^{1/4} \sim M^{-1}$ . Such branes will then interact via an exchange of the bulk particles. Some of these forces (e.g. gravity and dilaton) can give attraction, and others (e.g. bulk gauge fields) can give repulsion if branes have the same sign of charge.

Sometimes it may happen that, due to supersymmetry, there is a very precise (in perturbation theory) relation among brane tension ( $\sigma$ ) and its charge, so that the resulting force cancels out. In such a case the two-brane system is a BPS state and

there is a zero net force between them. This is, for instance, true for the parallel D-branes. In string theory picture this can be understood as vanishing of a tree level amplitude with closed string exchange, or alternatively of the one-loop open string amplitude stretched between the branes.

In an effective low energy field theory picture this can be understood as the graviton-dilaton attraction compensated by the RR repulsion.

However, in the real world supersymmetry is broken and we expect that the force is no longer zero. Then at very short distances  $r \sim M^{-1}$  the interaction between the low-energy modes that are localized on the different branes as well as the string modes that are stretched between the two become important and the sign of the potential depends on these couplings. We will assume that the overall interaction is repulsive. At the distances  $r \gg M^{-1}$ , however, these modes decouple and their contribution dies away exponentially fast. At large distances the brane interactions are governed by the two sources: 1) Exchange of the light bulk modes, e.g. such as the graviton, dilaton or RR fields; and 2) by the tension of the strings that are actually stretched between the branes. The second source provides a linear (in r) confining potential between the branes. However, its existence depends on a number density of such strings. It is very hard to produce them while branes are already separated. So such strings mostly will be important in the case when branes initially come close and get separated later. For the low energy observer these states will look as superheavy particles of the mass

$$m_{string} \sim M^2 r$$
 (2)

and can play the role of a dark matter. Below we will show that such states can be actually produced after the thermal brane inflation.

With the account of the above two sources, the brane interaction potential at large distances assumes the form:

$$V(r) = M^{4}(\alpha + b_{i} \frac{e^{-m_{i}r}}{(Mr)^{N-2}} - \frac{1}{(Mr)^{N-2}} + kr)$$
(3)

where  $\alpha, b_i$  are model-dependent constants of order one. k is proportional to the density of the stretched strings per unit brane-volume.

In our scenario the linear term even if presented initially will quickly redshift away during the first stage of inflation, but can be recreated after reheating, since branes sit on top of each other. So the initial density of the stretched strings will be given by the reheating temperature after the first inflation  $T_{in}$  (see below).

So to study a zero-temperature (zero particle density) behavior of branes, one can ignore the contribution of the confining energy and concentrate on the first term. The Yukawa type potential comes from the masses of the bulk modes and the inverse-power term comes from the gravitational interaction (for N=2,  $\ln(r)$  behavior should be understood). If some of the bulk gauge fields are lighter then M, then at the distances  $M^{-1} \gg r$  the attractive gravitational interaction can

be dominated by a repulsive gauge interaction and the over-all potential can be repulsive at the short distances. Thus, branes can experience the Van der Waals type interactions at zero T. In particular, this will be the case if the dilaton is "projected out" from the low energy physics. That is, if it gets a mass  $m_{dilaton} \sim M$  due to some non-perturbative strong dynamics. Let the mass of the remaining repulsive mode be m and for simplicity assume the only one such a mode. Then there is a stable equilibrium point at  $r_0 \sim 1/m$  with a zero net force. So at zero temperature branes will be stabilized at this point.

How small can m be? In the absence of a concrete model m can be naturally as small as the supersymmetry-breaking scale in the bulk, which if the SUSY-breaking occurs on a separate distant brane, can be all the way down to an inverse millimeter  $\sim 10^{-3} \text{eV}[2]$ . In what follows, we will keep m as a free parameter  $M \gg m$  and fix its value from cosmological constraints.

When the branes are at distance  $r_0$  the effective four dimensional cosmological constant is given by

$$\Lambda_{eff} = V(r_0) + \Lambda_{bulk} V_N \tag{4}$$

and must be canceled. This is the usual fine tuning problem on which we have nothing new to say. However, if the branes are brought at the distances  $r \to 0$  there will be an excess of the potential energy resulting into an effective cosmological term

$$\Lambda_{eff} = V(0) - V(r_0) \sim M^4 \tag{5}$$

At zero temperature, however, the repulsive potential at r=0 is steep. Thus, the branes will quickly relax towards  $r_0$  and no inflation will result. Below, we will show that the situation can change dramatically when temperature effects are taken into account<sup>3</sup>

## 3 Brane Stabilization from Inverted Hierarchy

From above discussion, it seems not unnatural to expect the Van der Waals type forces between the branes. The positions of the minima are then defined by the masses of the bulk fields, and the large distance stabilization would require some of these masses to be small. As said above, this smallness can be due to the smallness of the supersymmetry breaking scale in the bulk, which can be suppressed by a bulk volume factor  $\sim 1/(MR)^N$ . Stated in this way the issue becomes linked with a largeness of the extra dimensions. It is natural to ask whether branes can be stabilized at large distances, without help of the small parameters? This question has an independent motivation, since in such a case one can invert the issue and try to explain largeness of the radius by the large inter-brane separation.

<sup>&</sup>lt;sup>3</sup>The Van der Waals form of the potential can allow for the low reheating temperature even for the slow-roll brane inflation[17]. Here we will be interested only in the inflation driven by thermal effects.

In '81 Witten suggested a way generating a large mass scale from a small one as a result of the dimensional transmutation[16]. From the first glance the issue is not quite the same, since we are willing to generate a small mass scale instead of large. However, the duality between the infrared gravity and ultraviolet gauge dynamics suggests that the solution may work also here.

In the gauge theory description the brane separation is a VEV of the Higgs field, that gives masses to the open string modes. Thus, in this picture generation of large inter-brane distance is equivalent to generation of the large mass scale. Details will be presented in [22], here we will briefly discuss the main idea relevant for our purposes.

To illustrate the point, consider a toy N=4 supersymmetric example with a set of n D3-branes in space with two large transverse dimensions. When branes are coincident there is an enhanced SU(n) symmetry, which gets broken if some of them get displaced. In supersymmetric limit, the moduli space can be parameterized by an adjoint VEV  $\Sigma$ . Now imagine that some strong dynamics generates a superpotential for the lowest massless modes

$$W = \phi \Lambda^2 + \lambda \Sigma^3 + \dots \tag{6}$$

where  $\Lambda \sim M$  is some typical mass scale of this dynamics and  $\phi^2 = Tr\Sigma^2$ . At the tree level, this leaves N=1 supersymmetry among the massless modes, but we will assume that heavy modes do not experience breaking at this stage. What is the moduli space of this theory? If n is even, for  $\phi >> M$  there is a plateau along which supersymmetry is broken and the gauge symmetry is broken to  $SU(n/2) \otimes SU(n/2) \otimes U(1)$  at the scale  $\phi = M^2r$ , (where r is the brane separation). This means that at the tree level there is a net zero force between the branes if two sets of coincident n/2 branes are displaced. <sup>4</sup> However, because supersymmetry is broken, the one-loop corrections to the Kähler metric lift the plateau and generate the inter-brane potential

$$V(r) \simeq \Lambda^4 (1 + (a\lambda^2 - bg^2)) \ln(Mr) \tag{7}$$

where a and b are positive one-loop factors and g is a gauge coupling. This potential comes from the one-loop Kähler renormalization by the particles of mass  $M^2r$ . The heavier modes do not contribute because of (by assumption) higher supersymmetry. According to infrared-ultraviolet duality, the same asymptotic form should be recovered via the tree-level closed string exchange, which is indeed the case, since the long distance physics is dominated by the massless mode exchange, which give  $\ln(r)$  potential in the leading order.

 $<sup>^4</sup>$ As it stands, the superpotential in Eq(6) also admits the isolated supersymmetric minima with two sets of coincident n-k and k branes separated at distance  $r\sim \Lambda/M^2$ . In this respect this model is analogous to the model of [23]. This is unimportant for the present purposes, since we are interested in the long distance brane interactions. The Intriligator-Thomas type models[24] with no supersymmetric ground state can also be constructed. (See [25] for some brane model building in this direction)

Now the point is that  $\lambda$  and g should be understood as the running functions of  $\phi = M^2r$  and taking this running into account, the potential should get a "log-corrected" form

$$V(r) \simeq \Lambda^4 (1 - (c_1 - c_2 \ln(rM)) \ln(Mr)$$
 (8)

where  $c_1$  and  $c_2$  are of the order of one and two-loop factors respectively. So depending on the balance between parameters, this potential can have a minimum at  $ln(Mr) \sim c_1/c_2$ , stabilizing branes at very large distances. Of course, several things are not answered in this toy model. For instance, what about higher loop corrections, or how SUSY-breaking affects the open-closed duality? However, it illustrates the main idea. Some details are given in [22].

#### 4 Attractive Branes at High Temperature

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In high temperature supersymmetric gauge theories, the points of the restored gauge symmetries are always the local minima of the free energy. Since the coincident branes correspond to the enhanced gauge symmetry points, it is expected that branes get stabilized on top of each other at sufficiently high temperatures.<sup>5</sup>

Let us have a closer look at the nature of the zero-T repulsive potential of the branes at r=0. We can do this from the point of view of an effective field theory. In this picture relative displacement of the branes is described by an expectation value to the scalar field  $\phi=M^2r$ . In D-brane picture, when branes are on top of each other there is an enhanced gauge symmetry, which gets broken by  $\phi$  VEV when branes are separated. The gauge fields that get masses from  $\phi$  are the open string modes stretched between the different branes. The mass of the lightest modes from such a string will be  $\sim \phi$ . Now, in the unbroken supersymmetry limit, branes are BPS states and  $\phi$  is modulus with exactly flat potential. Appearance of the interbrane potential in this language corresponds to the lifting of the flat moduli space by supersymmetry breaking soft terms. In particular, the fact that branes are repulsive means that  $\phi=0$  point became unstable due to supersymmetry breaking, or in the other words,  $\phi$  got a negative soft mass. By dimensional grounds the curvature at  $\phi=0$  should be decided by the supersymmetry breaking effects on the brane:

$$V(\phi) = -\phi^2 m_s^2 \tag{9}$$

where  $m_s$  is the scale of supersymmetry breaking on the brane. For large distances  $\phi \gg M$  we should recover the usual inverse power (or  $\log(r)$ ) behavior

$$V(\phi) = M^4(\alpha + b_i \frac{e^{\frac{\phi}{m_i'}}}{(\phi/M)^{N-2}} - \frac{1}{(\phi/M)^{N-2}})$$
(10)

<sup>&</sup>lt;sup>5</sup>We will assume temperatures below the Hagedorn point. See, e.g. [26] for some recent discussions.

Although this form looks singular for  $\phi \to 0$ , singularity is just a reflection of the fact that some modes become massless at  $\phi = 0$  and must be "integrated in". This will smooth out singularity at the origin.

Now let us take into account the effect of the high temperature. For  $\phi = 0$  the string modes that get masses from its VEV are in equilibrium and their contribution to the free energy creates a positive  $T^2$  mass term for  $\phi$ , so that the resulting curvature term becomes

$$V(\phi)_T = (cT^2 - m_s^2)\phi^2 + \dots$$
 (11)

where c is a model-dependent factor. This effect will stabilize the branes on top of each other all the way down to a certain critical temperature  $T_c \sim m_s$  below which  $\phi$  gets destabilized and branes roll away from each other. It is crucial that  $T_c$  is set by  $m_s$  and not by M. Thus the vacuum energy density of the branes  $\sim M^4$  can dominate over the thermal energy  $\sim T^4$  and trigger inflation.

## 5 Thermal Brane Inflation

The resulting inflationary scenario is quite simple. We assume that there was a period of an early inflation with a high reheat temperature  $T_{in} \sim M$  at the end of which some of the repelling branes appeared to sit on top of each other and got stabilized by the thermal effects as suggested above. Once the temperature drops below  $T_{in}$  the potential energy takes over and the inflation results all the way until T drops below  $T_c$  and potential gets destabilized. Thus, the number of available e-foldings is given by

$$n_e = \ln(T_{in}/T_c) \tag{12}$$

Taking  $T_{in} \sim 10 \text{TeV}$  and  $T_c \sim 10^3 - 10 \text{MeV}$  we find a maximal possible number  $n_e = 10 - 15$  or so. This is enough to get rid of unwanted relics like bulk gravitons.

#### 6 Stretching the Superstring Dark Matter

Now, let us show that in this scenario the certain amount of a superheavy dark matter is expected to be produced in form of the superstrings that get stretched between the branes. $^6$ 

The possibility of producing a superheavy dark matter during a "conventional" inflation or preheating has been discussed in the literature[27]. The possibility of heavy particle production during the thermal inflation[28] is the closest four-dimensional counterpart of our scenario.

<sup>&</sup>lt;sup>6</sup>After this work was done, I learned from K.Benakli about his work in progress on a different possibility of using winding modes as a dark matter in high Planck scale scenario.

We suggest that in the brane picture there is yet another way of generating a superheavy dark matter in form of the sub-millimeter size superstrings. When branes sit on top of each other, the lowest modes from the strings that stretch between them are effectively massless and are in thermal equilibrium. Their starting number density is given by an initial temperature  $N_{string} \sim T_{in}^3$ . We will take  $T_{in} \sim M$ . As soon as temperature drops to below  $T_{in}$  the potential energy takes over and branes inflate. The string number density then drops exponentially fast  $\sim e^{-3n_e}$  and is  $N_{string} \sim T_c^3$  right at the end of inflation. After this point brane bound state gets destabilized and branes move away stretching the strings between them. In the field theory language this means that exited string modes are getting masses from the  $\phi$  VEV. These modes become non-relativistic and their number density freezes out within the time  $\sim m_s^{-1}$ . Let us take as an example the potential (3) with a single repulsive mode of mass m. Then, right after the end of inflation Universe is left with strings of the mass  $\phi_0 \sim M^2 r_0 \sim M^2/m$  and the initial number density  $N_{in} \sim T_c^3$ . The energy stored in this dark matter is  $\rho_{in} \sim T_c^3 M^2/m$ , which is a tiny fraction of the initial energy density of the oscillating branes  $\rho_{osc} \sim M^4$ . The brane oscillations reheat the universe to the temperature<sup>7</sup>

$$T_R \sim \sqrt{\frac{m_\phi^3}{\phi_0^2} M_P} \tag{13}$$

Where  $m_{\phi} \sim M(m/M)^{\frac{N}{2}}$  is the oscillation frequency, the mass of the oscillating inflaton field  $\phi$ . After this point the string energy density scales as  $T^3$  so that the present day abundance can be estimated as

$$\Omega_{string} = \rho_{string}/\rho_c \sim 10^9 \text{GeV} \frac{T_c^3}{mM^2} T_R$$
(14)

Now, from the graviton over-closure there is a strong bound on  $T_R[11]$ . which in the case of two extra dimensions gives  $T_R \sim \text{MeV}$ , even for  $M \sim 10 \text{TeV}$ . From (13) this gives  $m \sim 10 \text{MeV}$  or so. Taking this numbers, the right abundance  $\Omega_{string} \sim 0, 3$  could have resulted if  $T_c \sim 1 \text{GeV}$ .

#### 7 Bulk Gravitons

Let us now, discuss dilution of the bulk gravitons by the brane inflation. First let us note that gravitons produced after the final reheating are safe as far as  $T_R \sim \text{MeV}$ , which in turn puts constraint on m. Thus, the main problem are the gravitons produced at the initial stage of reheating, just before the brane inflation. Let us

<sup>&</sup>lt;sup>7</sup>For the crude estimate we will omit the model-dependent numerical factors, which otherwise may be important, e.g. due to large number of species, or due to loop suppression of the inflaton couplings.

estimate their present abundance. Their initial number density is[11]

$$\rho_{gr} = \frac{T_{in}^{N+5}}{M^{N+2}} M_P. \tag{15}$$

This will be further reduced by a factor  $\sim (T_c/T_{in})^3$  during the period of the thermal brane inflation, and subsequently by a dilution factor  $\sim (T_R/M)^4$  during the period of the brane oscillations. After reheating the graviton energy density scales as  $T^3$  and the condition for it to never dominate the Universe becomes

$$\frac{T_{in}^{N+2}}{M^{N+6}} M_P T_c^3 T_R < 10^{-9} \text{GeV}$$
 (16)

Assuming  $T_{in} \sim M$  this can be satisfied if  $T_c \sim \text{MeV}$  or so. Note however that this inequality is very sensitive to the value of  $T_{in}$  and thus can be accommodated even for larger  $T_c$  provided  $T_{in}$  is somewhat smaller than M.

# 8 Discussions (or the Role of the Bulk Vacuum Energy)

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We have discussed the issue of the infrared brane-brane stabilization which may have some important consequences.

First, the large distance brane-brane stabilization, may be responsible for the large size of the extra dimensions. For this, one has to assume that branes "decide" the size of the bulk volume, that is (in the leading order) the potential for the radius modulus comes purely from the brane-brane interactions. However, when branes get stabilized at large distances, usually, the potential is very shallow, the mass of the inter-brane mode is  $\sim 1/r_0$ . So one may wonder that the bulk vacuum energy can generate the stronger potential for r that would destroy this stabilization. For instance, a constant  $\Lambda_{bulk}$  term would generate a power-law  $\sim r^n$  potential. We want to stress that, in general, this is not necessarily the case if at the tree-level supersymmetry is broken on the brane and gets transmitted to the bulk only through some messengers like gravity. In such a case there can be a zero bulk vacuum energy to "start with". For a moment let us consider two extra dimensions (the role of these will become more clear on an explicit example below). Then, the naive dimensional analysis suggests that the integrated bulk vacuum energy should scale as

$$\sim M^4 f(\ln(\text{rM})) \tag{17}$$

where f is some (non-exponential) function of ln(rM). This naive argument relies on the fact that although the number of available bulk modes (lighter than the cut-off

<sup>&</sup>lt;sup>8</sup>We understand that the special role of the two extra dimensions was also pointed out in [15], [29] but from a different perspective.

scale) is enormous  $\sim (rM)^N$ , the supersymmetry breaking in each multiplet is tiny (suppressed by  $\sim (rM)^{-N}$ ). However, the things are somewhat more involved since the scaling law can depend how the SUSY-breaking scales with a distance from the brane. To illustrate the point let us consider a simple example with two transverse dimensions, which explicitly demonstrates how the log-scaling can appear. As it was suggested in [7], the supersymmetry breaking on the brane can simply result from the fact that the brane is a non-BPS soliton. The stability of the brane can be due to the topological charge. In particular, for two extra dimensions global vortexes can serve as non-BPS solitons[7]. Let us imagine a model with two dimensions compactified on a sphere  $S_2$ . Let  $\Phi$  be a complex scalar field defined on the sphere and we assume that  $\Phi$  has a non-zero expectation value  $\Phi = v$  due to whatever dynamics. Assume that  $\Phi$  transforms under a global U(1) symmetry  $\Phi \to e^{i\alpha}\Phi$ . Then, its expectation value breaks U(1) but in general leaves supersymmetry unbroken. Supersymmetry can be maximally broken if we discuss topologically nontrivial winding configuration (vortex) which (in a flat space limit) asymptotically look as[30]:

$$\Phi|_{\rho \to \infty} \to v e^{i\theta}$$
 (18)

where  $\rho$  is the distance from the core, and  $\theta$  is an polar angle. Consider a vortex-anti-vortex pair "stuck" at the opposite poles of  $S_2$ . The bulk vacuum energy of the system coming from the gradient term diverges logarithmically with vortex-anti-vortex distance r, which in our case sets the size of extra dimensions:

$$V(r) \sim \int_{\delta}^{r} \rho d\rho |\partial_a \Phi|^2 \sim v^2 \ln(rM)$$
 (19)

where,  $\delta$  is the size of the core. Thus, we recover the log(r) behavior. Note that, if it was only the potential energy of the vortex core, the supersymmetry would be unbroken at the tree level in the bulk. The logarithmic behavior of the bulk energy, can be understood as a result of a tree level transmition of the supersymmetry breaking from the brane to the bulk by the derivatively coupled massless Nambu-Goldstone field. The "local strength" of this breaking  $(\Lambda_{bulk})$  scales as  $\sim 1/\rho^2$  as function of the distance from brane so that integrated vacuum energy is  $\sim \ln(r)$ . In this example, Goldstone expectation value is a part of the winding configuration, however, in more generic case it may just play the role of a messenger at the loop-level. The above example demonstrates that the role of two extra dimensions can be crucial. Note that, for the three transverse dimensions with point-like branes the scaling could be different. For instance, for the global monopoles stuck on  $S_3$  the scaling would be linear in r.

Above discussion shows that it is not unusual for the bulk cosmological constant to have a log-scaling behavior and may not dominate over the direct inter-brane potential, which may be responsible for generating the large size of extra dimensions.

Finally, even if large inter-brane separation is not directly responsible for the large extra dimensions, it can still have a interesting cosmological application since can lead to a brief period of the thermal brane inflation with an acceptably low reheating temperature and superheavy dark matter.

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